Coalitional Politics and Logrolling in Legislative Institutions

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We examine how a foresighted legislative chamber will design its institutions in response to *ex ante* incentives for universalism and *ex post* incentives for minimum winning coalitions and what coalitions will form as a result. To do so, we develop a model of vote trading with an endogenous voting rule and coalition formation process. We find that in equilibrium, legislative chambers will almost never choose institutions that guarantee either simple majorities or universalistic outcomes. Rather, coalition sizes from minimal winning to universalistic will be possible under certain conditions given the choice of voting rule. Further, these coalitions will be "minimal necessary," just large enough to sustain cooperation.

Political scientists have long been aware that politicians have an incentive to construct minimum winning coalitions (MWC) for distributive policies (Riker 1962; Koehler 1972; Shepsle 1974; Uslaner 1975; Koford 1982; Denzau and Munger 1986; Baron and Ferejohn 1989, but see Baron 1989; Groseclose and Snyder 1996). Including additional members tends to raise costs without adding benefits to the coalition.

However, starting in the late 1970s, and at least partly in response to empirical evidence (Ferejohn 1974; Arnold 1979; Wilson 1986; and Collie 1988, among others[1]), political scientists also started to generate theories that predicted universal or oversized coalitions. For example, Weingast (1979) finds that, although games of distributive politics resulted in minimum winning coalitions, the electoral need to bring benefits back to the district and fear of being left out of the minimal coalition could cause politicians *ex ante* to prefer universalistic coalitions. As such, politicians would have an incentive to set up "norms of universalism" enforced, perhaps, by punishment for not going along with the all-inclusive logroll. Later work found that such universalism would be preferred even for inefficient pork barrel legislation (Shepsle and Weingast 1981; Weingast, Shepsle, and Johnsen 1981), as well as in political systems with committee structures (Fiorina 1981).[2]

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[1]All the empirical evidence does not point one way. For example, Stein and Bickers (1994) analyze the number of congressional districts receiving benefits from various spending programs. They find that, in addition to benefits not being spread universally, programs generally do not even benefit a majority of districts. Weingast (1994) calls these findings into question by raising issues such as complementarities and the role of parties.

[2]In an alternate framework, Niou and Ordeshook (1985) make explicit a game structure in which legislators, under certain conditions, might choose universalism over minimum winning coalitions as an optimal individual choice.

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While these studies argue persuasively that politicians want universalistic outcomes *ex ante*, and that institutional structure *can* encourage universalistic outcomes even when *ex post* incentives for MWCs exist, they do not prove that politicians *will* overcome their individualistic incentives when forming institutions. Thus, we want to examine how a foresighted legislative chamber *will* design its institutions in response to *ex ante* incentives for universalism and *ex post* incentives for MWCs, and what coalitions *will* form as a result.[3] Our study is a natural extension of recent work that asks: if institutions matter, how do politicians set up those institutions in the first place? For example, this question already has been asked with regard to committee structures (Gilligan and Krehbiel 1987, 1990; Krehbiel 1991), bureaucracies (McCubbins, Noll, and Weingast 1987, 1989), and legislative "hurdle factors" (Diermeier and Myerson 1998).

Here, we model distributive politics in a legislature allowing both an endogenous coalition formation process and an endogenous initial choice of the voting rule. Faced with the *ex ante* desires and *ex post* pressures, politicians choose the voting rule anticipating how it will affect future coalition sizes. We find that the choice of voting rule depends on the size of the legislature, on the ease of coalition formation, on reelection considerations, on the efficiency of expected legislation, and on the expected size of the coalition, which ranges between the minimum number dictated by the voting rule and universalism. The choice of coalition size depends upon all the same factors the voting rule depends on, as well as on the voting rule itself.[4]

In equilibrium, consistent with the *ex ante* desires to promote universalism, legislators choose a voting rule to ensure that as much legislation is passed as possible. However, because the game structure does not allow cooperation over distributive legislation to be guaranteed, universalism is not assured. In fact, the model predicts coalition sizes from MWCs to universalism, depending upon the previously listed factors. Conversely, consistent with the *ex post* desire to only form MWCs, proposers make coalitions "as small as possible." However, again because the model does not hardwire in cooperation, as small as possible is not necessarily a MWC. In fact, oversized coalitions are formed because MWCs often cannot sustain cooperation. Thus, while coalitions are minimum winning in the sense that they are as small as possible while still maintaining cooperation, they are not minimum winning in the sense of being equal to the number of votes necessary for passage given the voting rule. We define this new conception of minimum winning as a "minimum necessary coalition" (MNC).

We divide the paper into four sections. In section two we analyze a baseline logrolling game that characterizes some of the important underlying dynamics of our model. We set up an N-person legislature that is trying to engage in an infinite string of logrolls. Each logroll consists of a sequence of votes over a set of bills in which each bill benefits one member of the legislature at the expense of all other members. We find that it becomes increasingly difficult to maintain the logroll as the size of the chamber increases and as the voting rule becomes more inclusive.[5] Further, we show that the voting rule chosen by the chamber is the most inclusive one that still permits cooperative logrolls, given the chamber size. We present this version of the model to demonstrate the logic of the game structure and to generate closed-form solutions. All derived relationships hold across both the baseline and fully specified models.

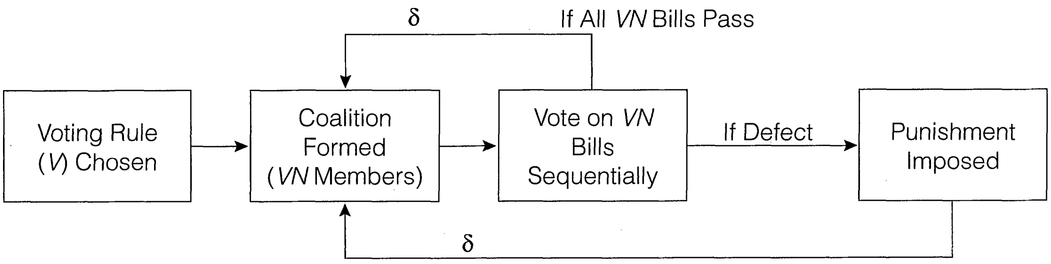
In section three we specify the full model, allowing for a variety of bill types, a random proposer, and coalition sizes larger than minimum winning. We solve the game implicitly for the parameters of interest and characterize comparative statics through a simulation. The equilibrium voting rule results are consistent with the baseline model. Further, by allowing both the bill type and the coalition size to vary, we can fully explore how the coalition formation processes work. One of the main results is that oversized coalitions are able to sustain cooperation where minimum winning coalitions cannot. As such, the same conditions that give rise to pressures to decrease the voting rule also bring about larger oversized coalitions. That is, given the voting rule, if either the actual cost of legislation increases, the chamber size increases, or the value that legislators place on the future decreases, then the coalition size increases. Thus, taken together, these results paint a complex picture of how decision-making bodies respond to changing membership, bill makeup, and other characteristics, with changes in their coalition sizes and voting rules. Section four offers ideas for future extensions of this work, and section five concludes.

[3]For other work on how institutional constraints affect coalition processes, see Binder and Smith (1996), Brady and Volden (1998), and Krehbiel (1998).

[4]In a sense, this work is related to that of Buchanan and Tullock (1962). They explore the question of why different constitutions specify different voting rules. In their model, more inclusive voting rules (ones closer to unanimity) entail higher negotiating costs, but also increase protection from adverse decisions. Thus, in choosing their voting rules, members are trading off protection from external costs for a lower cost of decision making.

[5]We characterize voting rules as lying on a continuum from least inclusive (simple majority rule) to most inclusive (unanimity).

FIGURE 1 Sequence of Game Actions



The Basic Model

We explore a baseline model that allows for an endogenous choice of voting rules, but restricts the coalition size to MWC. This specification allows us to focus on the role that variables other than the coalition size play in the model, including the legislature size, the cost-benefit ratio, and the ease of forming coalitions. The model is an infinitely (or, indefinitely) repeated game with N legislators in a legislature. Each legislator seeks to maximize the total net present value she receives from the passage of all current and future legislation.

Two assumptions are made over the structure of all legislation for this baseline model. First, all bills are identical with the exception of who is the beneficiary. Second, each bill has some benefit, B, and cost, C, where the benefits from a particular bill go to a single actor, and the costs are evenly dispersed among all actors (C/N is paid by each actor for each bill passed). Thus, these are particularized bills in which, if actor i's bill passes, that legislator receives $B - C/N$ and all other actors receive $-C/N$. This payoff structure is typical of distributive legislation, such as public works projects, in which some project funded through federal taxes is built in a legislator's district. Note, legislation will only pass when it is net social welfare improving, $B > C$. Given a fixed, known cost, any chamber choosing its voting rule when $C > B$ would trivially set the rule at a level that ensured nothing would pass. Thus, in this baseline model we only discuss the case of $C < B$.

The game, as illustrated in Figure 1, consists of three main stages: institution building, coalition formation, and legislative. During the institution building phase the chamber picks a voting rule ($1/2 < V \leq 1$).[6] To capture the universalism literature's supposition that *ex ante* politicians want universalistic outcomes, we assume that the legislators act behind a veil of ignorance. That is, all legislators are defined to be identical *ex ante*. While this assumption makes the decision rule in this stage irrelevant, one can assume for concreteness that the voting rule is passed by unanimity.[7] Once the voting rule is established, it is fixed for the rest of the game.[8]

The second stage, coalition formation, is the first of the two repeated stages of the game. In it, the various actors engage in a search process, trying to find a mutually beneficial logroll. Formally, this process is represented by nature randomly picking a minimum winning coalition from the pool of available actors (those in "good standing," to be defined later) and putting their legislation on the docket. Again, these assumptions are made for expositional purposes and are relaxed later, when we allow a proposer to select coalition size and membership.

Once the coalition's bills are on the docket, legislators vote on them sequentially. Bills pass if they receive at least as many votes as required by the voting rule and fail otherwise. We explicitly do not allow bills to be bundled into a single piece of legislation because we wish to allow the possibility for coalitions to unravel. This helps provide the *ex post* incentives for minimum winning coalitions that we wish to study. After the last vote, punishment is meted out as necessary, and the game returns to the coalition formation stage.[9] Each time the game re-

[6] We restrict the voting rule to be at or above majority rule. Similar results follow from allowing less inclusive voting rules, but such voting rules have peculiar properties (i.e., if a majority opposes some legislation and yet it passes, that majority could simply pass legislation repealing the earlier changes) and rarely exist empirically.

[7] An interesting extension of this basic model would allow the legislators to have priors over their probability of being in the first, or early, coalitions. This set-up would reflect the idea that a chamber could know who is likely to benefit from upcoming legislation during the institution-building stage. While this extension would be interesting and may be addressed in future work, we do not explore it here since these priors create incentives for politicians to not pursue universalistic goals *ex ante*.

[8] Another interesting extension would allow the voting rule choice to be revisited by legislators as the game progresses. This possibility is discussed in the extensions section.

[9] Note that we do not allow the legislative chamber to reconsider bills in a future period. Allowing such behavior would reduce (but not eliminate) incentives to defect.

turns to this stage all future payoffs are discounted by δ, where $\delta \in [0,1]$. Discount rates typically reflect the diminished value of the future and depend on the length of time associated with moving from one period to the next. Here, this discounting also can be interpreted to represent the delay associated with finding a new coalition or the frequency of elections and probability of losing office. The less conducive the given institutional structure is toward finding new stable coalitions the longer it takes for the new coalition to form, and thus the smaller the discount value. The more frequent elections and lower the reelection rate, the less likely the legislator will enjoy future utility, and therefore the smaller the discount value.[10]

With the model structure in place, all that remains is to characterize the punishment strategy and solve the game. As in all infinite horizon games, there is a multitude of possible punishment strategies available, and those strategies can yield an infinite number of Nash equilibria. According to Kreps (1990), the only hold that any Nash equilibrium should have on us is that it helps find the "obvious" way to play a game. We use a punishment strategy we call *parole* because: (1) it could be employed by an average politician and (2) it fits Kreps' criteria.[11] This punishment strategy has the following characteristics. If a member of the coalition defects and votes against another member's bill, the defector is placed in "bad standing." No one will choose to form a coalition with that individual for the "foreseeable future" because of his reputation of being untrustworthy. The legislative process goes on with only the members in good standing forming coalitions and passing bills. However, if enough members of the chamber have defected such that the remaining members in good standing do not have enough votes to pass legislation, they will again include the member who first defected. Thus, all this strategy requires is that each member of the chamber remembers if and when a person went into bad standing and for individuals to not trust defectors. [12] Further, it allows the chamber to continue passing legislation despite defections.

To see how the punishment strategy works, imagine a chamber with five members and a three-fifths voting rule. Suppose in the first round of the game the coalition, randomly consisting of members one, two, and three, is betrayed by player one after his bill has passed. Players two and three do not get their legislation passed, and a new coalition is formed. At this point player one is ostracized and cannot become a member of any future coalitions. The game continues indefinitely, with coalitions forming among players two through five, until someone else, say player two, defects. Now both players one and two are in bad standing. As a result, players three through five deterministically comprise all future coalitions until a new defection occurs, at which point that defecting player is considered in bad standing and is replaced by the noncoalition member who defected longest ago (i.e., player one in this example). Likewise, upon another defection, player two would be allowed back in the coalition. And so on.[13] As in most repeated games, actual punishment will be found to be off the equilibrium path.

Note that when the voting rule is unanimity, parole would let a defector immediately back into good standing and thus provide no incentive to cooperate. As a result, in the special case where $V = 1$, the punishment strategy is grim trigger (i.e., a defector is out of good standing forever). Other strategies, such as a T-period punishment, could be used as well.[14]

The Logrolling Subgame

Identifying when cooperation is sustainable over a logroll requires determining who has the greatest incentive to defect and then calculating the conditions under which that person will cooperate. In this model the legislator with the greatest incentive to defect is the person with the first bill in the logroll. This first voter can vote with the coalition on his bill, and defect thereafter, thus gaining the benefit of receiving his own bill's benefits while not having to pay for any other bills this period. If cooperation is to be sustained, these benefits from defection must be outweighed by the punishment.

In the unanimity case, the cooperation payoff is the net present value of maintaining logrolls infinitely into the future, $\frac{B-C}{1-\delta}$, and the defection payoff is the return

[10]Another possible extension would allow δ to vary by individual, as the probability of any individual being reelected.

[11]This strategy also turns out to be maximal in the baseline model because it places the greatest possible punishment threat on the individuals most likely to defect.

[12]We only consider simple punishment strategies because, as proven by Abreu (1988), all perfect equilibrium paths in an infinitely-repeated game with discounting can be characterized as the outcome of a strategy profile which is independent of history (i.e., "simple").

[13]Since we do not explicitly model the coalition formation stage, the punishment strategy is partly built into the game structure (in the form of being left out of the pool of legislators from which nature draws future coalitions). However, as shown below with a random proposer, this simplification does not drive the equilibrium results.

[14]In particular, a renegotiation proof T-period strategy in which the defector engages in his/her own punishment provides equivalent results.

from having just one's own bill passed this period and none ever again, $B - \frac{C}{N}$. Cooperation is sustainable as long as

$$\delta \geq \delta_U^* = \frac{C - C/N}{B - C/N}. \quad (1)$$

See Appendix A for proof.

In the nonunanimity rule case, legislation is passed if *VN* members of the legislature vote for the bill, where *V* is the voting rule (e.g., 1/2, 2/3, etc.), and *N* is the size of the chamber. If the first individual cooperates, he will indefinitely receive a benefit of *B* whenever he is in the coalition and will pay for *VN* bills each period. If he defects, he will receive *B* this period, without having to pay for the others' bills, but will not have a chance of being chosen for future coalitions because of being in bad standing (it is off equilibrium path for anyone else to defect in the future). As shown in Appendix B, the first person's payoff from cooperation exceeds the payoff from defection as long as:[15]

$$\delta \geq \delta_{NU}^* = \frac{VC - C/N}{VB + VC - C/N}. \quad (2)$$

For this equilibrium to be subgame perfect, we must show that the punishment would be carried out and that cooperation is preferred to defection given the punishment strategy. That is, members in good standing must not want to allow members in bad standing to become part of future coalitions; members in good standing must not want to selectively exclude other members in good standing; and cooperation over logrolls (given there are members in bad standing) must be maintained. Starting with the last constraint, if $\delta \geq \delta^*$ cooperation is sustained in any subgame with any number of members in bad standing. Cooperation always is easier to maintain when there is an enhanced probability of being in future coalitions, and each additional member in bad standing increases the chances of being in a future coalition for those in good standing. Second, and by implication, the members in good standing always want to exclude the members in bad standing since members in good standing are strictly better off with the higher probability of being in future coalitions. Finally, a proposer's future payoff is independent of which members in good standing join the coalition, so there is no incentive for a proposer to selectively exclude other members in good standing. Therefore, this equilibrium is subgame perfect.

[15]Inserting $V = 1$ into Equation 2 does not yield Equation 1 because the parole punishment strategy allows punishment through future bill passage that is not possible with grim trigger.

These solutions for the discount rate (δ) show how the model's parameters affect the possibility of cooperation. First, as *N* increases, δ^* increases. That is, as the chamber size increases, cooperation is more difficult to maintain. Second, as *C/B* increases, δ^* increases. Increasing the cost-benefit ratio makes cooperation more difficult to sustain. Finally, there is a positive relationship between *V* and δ^*. That is, as voting rules become more inclusive, a smaller range of cooperative logrolls can be sustained. In sum, it is easier to maintain cooperative coalitions for logrolls where: the number of legislators is small, the bills are much more beneficial than costly, the future is highly valued, the probability of reelection is high, coalitions can be formed quickly and easily, and voting rules are less inclusive.

Next we ask how the size of the legislature, the type of legislation (i.e., relative costs and benefits), and the ease of coalition formation affect the choice of voting rule.

The Institution-Building Supergame

In choosing the voting rule, the actors have no priors over whether they will be in the first, or any succeeding, coalitions. They only know that there are some given costs (C) and benefits (B) associated with each bill and that other parts of the chamber's institutional structure determine how easy it will be to form new coalitions (δ). With this limited information, the actors want to choose a voting rule that maximizes their expected benefit from playing the game. The expected utility given no defections is $EU = \frac{V\delta}{1 - V\delta}(B - C)$, as each period *VN* bills are expected to pass with benefits *B* and cost *C*. Since all bills are "good ones" ($B > C$), this expected value is increasing in the number of bills passed, and the chamber will always agree unanimously upon the rule that generates the largest logroll but still permits cooperation (as detailed in the previous section).

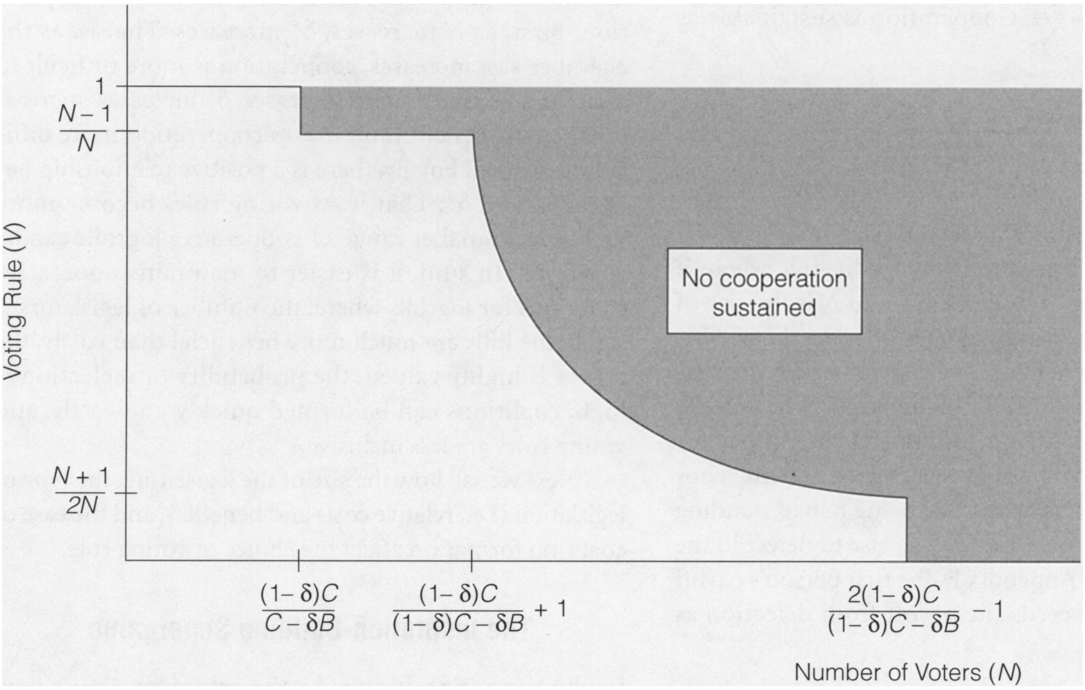
For the nonunanimity case, the upper boundary on the voting rule can be made explicit by solving Equation 2 for *V*:

$$V \leq V^* = \frac{C}{N\left(C - \frac{\delta}{1 - \delta}B\right)}. \quad (3)$$

We can now see how the model's parameters affect voting rule choice. Figure 2 illustrates how chamber size affects the choice of voting rule in equilibrium.

The horizontal axis measures the chamber size, and the vertical axis measures the voting rule. The shaded region demarks where cooperation is unsustainable, the

FIGURE 2 Voting Rule Chosen (by N)



clear region demarks where cooperation is sustainable, and the solid line is the upper boundary of the cooperative region indicating the voting rule that is chosen in equilibrium. This graph has four main regions. In the first region, for the number of legislators (N) less than $\frac{(1-\delta)C}{C-\delta B}$, cooperation can be sustained under unanimity.[16] The chamber is small enough to allow cooperation to be sustained even under the standard grim trigger punishment strategy. In the second region, between $\frac{(1-\delta)C}{C-\delta B}$ and $\frac{(1-\delta)C}{(1-\delta)C-\delta B}+1$, the chamber size is small enough to allow cooperation at the maximally inclusive supermajority rule $\left(V=\frac{N-1}{N}\right)$, but not unanimity.[17] This plateau exists because the maximal punishment under nonunanimity rules is harsher than that under unanimity. In the third region, between $\frac{(1-\delta)C}{(1-\delta)C-\delta B}+1$ and $\frac{2(1-\delta)C}{(1-\delta)C-\delta B}-1$, the voting rule monotonically decreases as chamber size increases.[18] Intuitively, the closer to a simple majority rule, the fewer pieces of legislation being cobbled together and therefore the lower the costs that can be avoided by defecting. Conversely, the larger the chamber size, the greater the costs that can be avoided by defecting. Thus, when an increase in size pushes the chamber into the noncooperation region, the voting rule must become less inclusive. Finally, in any chamber larger than $\frac{2(1-\delta)C}{(1-\delta)C-\delta B}-1$, logrolls are unsustainable. The temptation to defect and save the costs associated with passing the rest of the logroll is too large. These findings yield the following proposition.

Proposition 1: As legislature size increases, voting rules must be made less inclusive to sustain vote trades.

This proposition leads to the prediction that we would expect to see small legislative bodies having more inclusive voting rules than larger bodies. For example, it makes sense that the U.S. Senate has an inclusive cloture

[16]This boundary condition is found by solving Equation 1 for N.

[17]The upper boundary on N can be found by inserting this inclusive voting rule $V=\frac{N-1}{N}$ into Equation 3 and solving for N.

[18]The upper boundary on N can be found by inserting the majoritarian voting rule $V=\frac{N+1}{2N}$ into Equation 3 and solving for N.

rule to end filibusters while the House does not even allow filibusters. Senators value their ability to influence legislation as part of a minority group or as individuals (Binder and Smith 1996), but such a power in the larger House would lead to even less cooperation and deference than we see today. Legislative bodies that expand over time likewise may need to loosen their voting rules. As the European Union expanded its membership, the Council of Ministers abandoned its unanimity rule to adopt a strict qualified majority rule, which may be loosened again if the EU goes ahead with a further expansion (Carrubba and Volden 1999).

Figures similar to Figure 2 could likewise illustrate the changing voting rules associated with the model's other parameters. For example, graphing the chosen voting rule for various discount values would yield four similar regions to those in Figure 2, but in reverse order. For high values of δ, the legislators value the future enough to allow logrolls to pass under unanimity rules. As the discount value decreases, however, it takes the stricter punishment strategy of parole to bring about cooperation under near unanimity. For even lower values of δ, the voting rule must be made less and less inclusive, until the point at which even a simple majority cannot be gained to support the logroll. This is reflected in the following proposition.

Proposition 2: Legislators prefer a less inclusive voting rule when coalition formation becomes more difficult, when the future is less highly valued, and/or when elections are more frequent and more hotly contested.

As we noted above, the discount value (δ) could be thought of as a parameter indicating the ease of coalition formation, the frequency at which logrolls are secured, the amount that legislators value the future, or the probability of reelection. If logrolls are easy to form and occur frequently, legislators will care more about being involved in future vote trades (δ will take on a high value). Because they care strongly about their chances of being included in the numerous future coalitions, legislators are more willing to cooperate with present logrolls, even those that include large supermajorities. Were coalitions more difficult to form and logrolls less frequent occurrences, cooperation could not be sustained under strict voting rules. As such, legislators would push for less inclusive rules. Likewise, if coalitions were more easily formed, the value of future interactions increased (as due to legislative professionalization), elections were less frequent, or legislator turnover was lower, legislators would like to see more inclusive rules that would guarantee benefits for more legislators while still allowing cooperation.

The final set of parameters influencing whether or not logrolls can be sustained, and in turn influencing the choice of voting rules, is the cost-benefit ratio of the bills being considered. As can be seen by looking at Equation 3, as costs increase or benefits decrease, V decreases. This finding yields proposition 3.

Proposition 3: As the cost-benefit ratio increases, the voting rule must be made less inclusive to sustain cooperation.

The larger the cost-to-benefit ratio, the more the first legislator on the docket must pay before having a chance at future benefits that are worth relatively less. Thus, for a given voting rule and discount value, coalitions will hold together for bills that are sufficiently more beneficial than costly, but will fall apart for bills that are not (e.g., where C/B is large). As such, during times when government projects are not considered worthwhile (due to high taxes, large deficits, or little public outcry for governmental programs) we should expect to see coalitions falling apart, governmental gridlock, and pleas for less inclusive voting rules.[19]

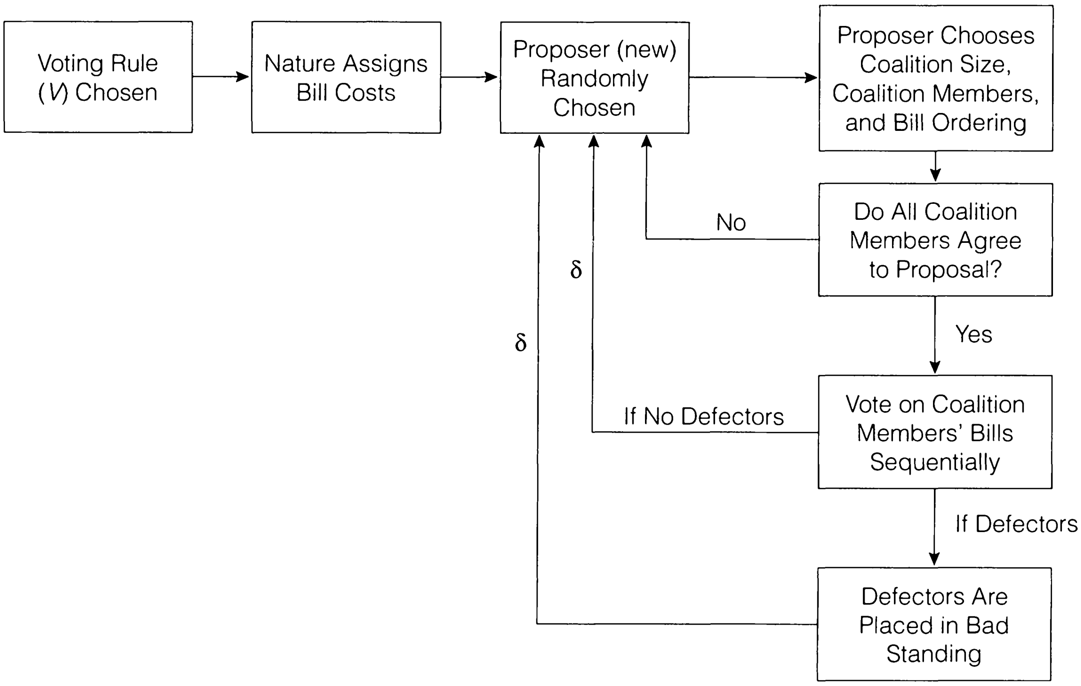
With the basic intuition in place, we turn to a more explicit model of the coalition formation stage of our model, demonstrate our basic results' robustness, and show under what conditions oversized coalitions and universalism will arise in distributive politics. We also briefly discuss some legislative efficiency implications.

The Full Model

The overall game structure follows the form of the original model. The legislators agree upon a voting rule, a coalition is formed, the bills of the coalition members are voted on sequentially, punishments are imposed, and a new coalition formation stage begins. However, unlike in the baseline model, the full model endogenizes the coalition formation stage and allows bill costs to vary. Also, by necessity, the punishment strategy is modified to account for possible behavior in the coalition formation stage. These modifications are made so that we can derive equilibrium coalition size, as well as affirm that the results from the baseline model are robust to endogenous coalition formation and uncertainty in what future legislation will look like. The new model is illustrated in Figure 3 and discussed below.

[19]Brady and Volden (1998) draw a somewhat similar link between deficit budgetary politics and policy gridlock. Binder and Smith (1996) call for an abandonment of the filibuster in the face of gridlock.

FIGURE 3 Sequence of Expanded Game Actions



In the baseline model, coalitions are minimum winning sets chosen by nature from the pool of legislators in good standing. Here, these assumptions are relaxed, and instead a random legislator, from the entire pool of legislators, is selected to propose the coalition size and makeup. If that coalition does not agree to form unanimously, then a new proposer is selected. This process continues until a coalition successfully forms, or until all members fail to create a coalition. This change allows us to explore the conditions under which oversized coalitions form.

The bills under consideration also differ in the full model. While in the baseline model all bills had a fixed cost and that cost was known prior to selecting the voting rule, in the full model bill costs are uncertain. At the beginning of the game, prior to the voting rule being selected, the actual cost of legislation is unknown. The legislators only know that there exists a common cost of legislation that will be drawn from an underlying distribution, here a uniform distribution between 0 and $\overline{C}$, after the voting rule is in place. We assume $\overline{C}$ is larger than B, so legislation can be net costly. Once the voting rule is fixed, the actual cost is drawn and that cost is used for the rest of the game.[20] This change allows us to explore how a voting rule will be chosen given an inability to know with certainty what future legislation will look like.

With the change to the coalition formation stage, there also must be a slight modification to the punishment strategy. Recall, in the baseline model, the punishment strategy was partly hardwired into the model because we assumed nature selected a minimum winning coalition from the pool of people in good standing. Now that these assumptions have been relaxed, the punishment strategy must describe what happens if a coalition is formed with a member in bad standing. Here we use a simple, intuitive solution: any legislator who enters a coalition with a member in bad standing is placed in bad standing as well. Simply put, an ally of an enemy is treated as an enemy. Beyond that minor modification, the rest of the punishment strategy is identical to the baseline version.[21]

[20]All results hold if we assume two cost types that are allowed to vary across periods or individuals, as well as for Nash solutions when costs are drawn each period from a uniform distribution. Subgame perfect solutions when costs are drawn each period from a uniform distribution are intractable, although there is no reason to believe that they will differ from the Nash solutions. Proof is available from the authors upon request.

[21]This strategy is renegotiation-proof under all equilibrium voting rules except unanimity. If the voting rule is unanimity all parties are better off choosing to forgive a person in bad standing than punishing him forever and never getting legislation passed again. The simple solution to this problem would be to make the punishment strategy a T-period punishment with the defector participating in his/her own punishment. And again, the only change would be in the cutpoint at which cooperation is no longer achievable.

Analysis

The steps necessary for solving this game are basically the same as in the baseline model. First, cooperation over the logroll once again depends upon whether the person with the first bill on the docket wants to cooperate. If that person wants to cooperate, all members of the logroll want to cooperate. Conditional on this constraint, the randomly chosen proposer wants to make a coalition proposal that is just large enough to sustain cooperation over the logroll. Adding unnecessary members to the coalition only reduces the net benefit to the proposer, while suggesting a coalition that is too small to sustain cooperation cannot yield a positive payoff. Finally, the voting rule is chosen anticipating what coalitions will form, and therefore what logrolls will occur, given any possible cost. Once more, because all members are identical *ex ante*, they will unanimously choose the voting rule that maximizes the expected social welfare.

In the following subsections, we analyze how these incentives play out. Appendices C and D present the formalization of this model.

Cooperation under minimum winning coalitions. As derived in Equation 2, cooperation in a MWC is possible as long as $\delta \geq \delta^* = \dfrac{VC - C/N}{VB + VC - C/N}$. Thus, solving this for C establishes the level of costs under which cooperation can be sustained:

$$C \leq B\left(\frac{\delta}{1-\delta}\right)\left(\frac{VN}{VN-1}\right) \equiv C_\alpha. \tag{4}$$

As long as the costs drawn for the legislation are less than this C_α, a minimum winning coalition can be maintained. This solution is detailed in Appendix C1.

Cooperation under oversized coalitions. Interestingly, cooperation is *easier* to sustain given oversized coalitions than given minimum winning coalitions. To see why, imagine that a coalition forms that is MWC plus one. In this situation the person with the first bill on the docket makes a very different calculation than when the coalition is just MWC. If only the first person defects, he goes in bad standing and all the rest of the bills still pass. The only way the logroll fails is if two people choose to defect. Thus, the only time the first person chooses to defect is when he knows that someone else intends to defect as well. In other words, defection becomes a coordination game.

This coordination game makes defection *less likely*, because if only one of them defects, the expected value for the other of being in good standing increases. That is, in the minimum winning coalition the potential defector is evaluating whether it is better to cooperate and play the same game next period or to defect and save the cost of the rest of the $VN - 1$ bills. In an oversized coalition, however, the potential defectors are evaluating whether it is better to cooperate and play a game next period in which all the others who defected are in bad standing, or to defect also and save the cost of the rest of the $VN - 1$ bills. The more oversized the coalition, the stronger the incentive to be the one who does not defect, because that will leave more members in bad standing and thereby increase the benefit of being in good standing (i.e., make it more likely to be selected to be in future coalitions).[22]

More formally, as costs rise above C_α, an additional A member must be added to the coalition if it is going to sustain cooperation. Appendix C2 shows that cooperation can be sustained if and only if:

$$A \geq A^* = N - \left(\frac{\delta}{1-\delta}\right)\left(\frac{VN}{VN-1}\right)\left(\frac{BN}{C}\right). \tag{5}$$

Of course, even oversized coalitions are limited by the size of the legislature. For some level of costs, even a universal coalition cannot bring about cooperation. As detailed in Appendix C2, universalistic cooperation breaks down when:

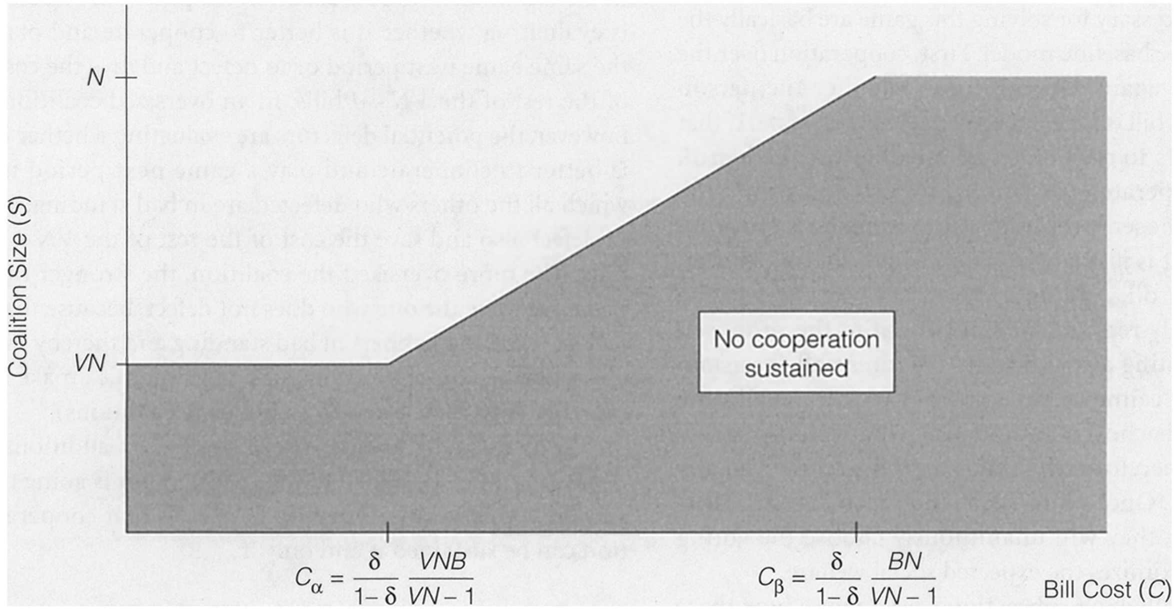
$$C \geq \left(\frac{\delta}{1-\delta}\right)\left(\frac{NB}{VN-1}\right) \equiv C_\beta. \tag{6}$$

As above, the solution concept used is subgame perfection. In subgames with some legislators in bad standing, once again the proposer chooses the smallest coalition that will allow cooperation. Most crucially, the *identical* conditions that make $VN + A$ legislators cooperate when all are in good standing *also* make a MWC possible when A legislators are in bad standing. This is the subgame equilibrium needed to solve the game, as detailed more fully in Appendix C2.

Choosing coalition size and membership. Given the conditions under which cooperation is sustainable, the randomly chosen proposer chooses a coalition with the following properties: (1) she is a member of any proposed coalition; (2) the coalition is the smallest one that can sustain cooperation; and (3) it only includes members in good standing. She includes herself since that is the only way she can gain a benefit; she chooses the smallest coalition because including additional members only creates

[22]This equilibrium solution relies on the standard coalition-proofness refinement.

FIGURE 4 Equilibrium Coalition Sizes



additional costs; and she chooses only members in good standing because there is no benefit, and some cost, from adding members in bad standing. We call a coalition that is the smallest necessary to sustain cooperation a "minimum necessary coalition" (MNC). The important difference between MNC and MWC is that MNC sometimes is larger than MWC because MWC cannot always sustain cooperation. That is, MNCs sometimes have to be oversized.

The equilibrium MNC is presented in Figure 4. The horizontal axis indicates the realized cost and the vertical axis indicates the equilibrium coalition size. For costs between zero and C_α, a minimum winning coalition forms (size $S = VN$). In this region, the cost of paying for the rest of the logroll is not high enough to cause the person with the first bill on the docket to want to defect. Since the proposer prefers as small a coalition as possible, she therefore proposes minimum winning, it is accepted, and the logroll succeeds.

For costs between C_α and C_β, increasingly large, oversized coalitions will form. In this region, a minimum winning coalition is not sustainable because the person with the first bill on the docket would choose to defect rather than pay for the rest of the bills on the docket. However, by adding some number of extra members to the coalition, cooperation can still be sustained. The minimum number of additional members is characterized by Equation 5 above.

For costs greater than C_β, no coalition forms. That is, for high enough costs even universal coalitions cannot sustain cooperation. These findings, as derived in the appendix, described above and illustrated in Figure 4, yield the following proposition.[23]

Proposition 4: For a given voting rule, legislature size, and ease of coalition formation, coalitions will be minimum winning for low cost-benefit ratios, oversized for moderate cost-benefit ratios, and unsustainable for high cost-benefit ratios.

Similar propositions can be generated for the occurrence of oversized coalitions given changes in the ease of coalition formation, in the legislature size, or in the voting rule. They characterize the landscape of minimum winning, universalistic, and oversized coalitions under different conditions.

[23]Interestingly, if we fix the voting rule and allow bundling, for example, via an omnibus bill, the *ex ante* expected value of the legislative process almost surely decreases. Not only would there be no need for oversized majorities, but it would become easier to pass inefficient legislation. Thus, beneficial legislation that previously would have passed no longer would and some inefficient legislation that previously would not have passed now would. By implication, bundling may actually be a poor tool for selectively maximizing cooperation over net beneficial legislation. This finding might well explain why we observe the extensive use of germaneness rules. Of course, if the voting rule and bundling were simultaneously endogenized in this stylized setting, unanimity would always be chosen and all efficient legislation would pass. Thus, since such a result is clearly pathological, a more complete analysis of bundling is left for future research.

Choosing the voting rule. As in the baseline model, we can derive the equilibrium-voting rule by finding the choice that maximizes the legislators' expected utility. However, in this setting the calculation is more complex because the cost of the legislation is unknown to the legislators prior to setting the voting rule. The cost could be low enough to allow minimum winning coalitions to be sustained, high enough to bring about oversized coalitions, or so high that no legislation could pass in equilibrium.

This uncertainty forces legislators to face a tradeoff in choosing the voting rule. If the cost is likely to be low, between 0 and C_α, in Figure 4, legislators will want to select an inclusive voting rule to make sure as many low-cost bills are passed as possible. However, as Equations 4 and 6 demonstrate, as V increases both C_α and C_β decrease. Thus, while increasing the voting rule raises the value of the low cost coalitions, it limits the range of coalitions over which cooperation can be sustained. The optimal choice of $V = V^*$ that maximizes the expected value of all possible coalitions given the distribution of C is characterized by Equation 7 below (see appendix D for the derivation).

$$-\ln(V^*) = N - V^*N + 1 - \frac{\delta}{1-\delta}\,\frac{2N^2 - V^*N^2 + 2V^*N - N}{2(V^*N - 1)} \quad (7)$$

Due to the transcendental nature of this function we cannot solve explicitly for V^*. Instead, the following results come from a computer simulation. The computer simulation does the equivalent of inserting values into Equation 7 for broad ranges of each of the variables of interest and solves for V^*. We are then able to determine the comparative statics results. Table 1 displays how the voting rule varies given different legislature sizes and different discount values.

Table 1 Example Simulation Values for the Equilibrium Voting Rule

δ	V	N	V
up to .11	.5	3	.78
.13	.521	8	.572
.15	.558	13	.509
.17	.6	18 and on . . .	.5
.19	.648		
.21	.705		
.23	.78		
.25	.928		
.26 and on . . .	1		

Note: For the left-hand table we fix $N = 3$ and for the right-hand table we fix $\delta = .23$. The comparative statics are most pronounced in these cases. See the text for explanation.

As Table 1 illustrates, the results are basically the same as those of the baseline case. As δ decreases or as chamber size increases, the voting rule becomes less inclusive. Once again, the smaller δ and the larger the chamber size, the greater the incentive for the person with the first bill on the docket to defect. The coalition size, in equilibrium, is chosen just large enough to ensure that the first person does not want to defect. Since the proposer wants the logroll to succeed and knows what the actual cost of legislation is, endogenizing the coalition formation stage and allowing costs to be uncertain when the voting rule is chosen does not change these underlying relationships. Thus, Propositions 1–2 hold once again in the expanded model.[24]

There is one additional finding of the full model results that should be highlighted. As N gets larger, the relationship between the voting rule and δ quickly approaches a binary setting. That is, even as soon as N reaches 18, for δ less than .23 the equilibrium voting rule is simple majority and for δ greater than .24 the equilibrium voting rule is unanimity. This limiting condition is reached so quickly because each time we add a legislator to the chamber we are also adding a new, unique bill. If we allowed chamber size to increase without requiring each person to have an individualized bill, the limiting case would occur much more slowly.

To characterize the other extreme, imagine that there were three bills under consideration and, each time chamber size increased one more person interested in each of the three bills was added. Under these conditions, increasing chamber size would have no impact upon the equilibrium-voting rule, because only those three bills would ever be under consideration. Thus, how quickly the limiting condition is reached depends upon how particularized the legislation is. Another way of stating this finding is that an increase in chamber size only matters to the degree that it correspondingly adds new dimensions to the policy space.

A Note on Efficiency Considerations

Much recent work on distributive politics and logrolling has focused on whether inefficient legislation, defined as $B < C$, can pass in equilibrium (Baron 1991, among others, define efficiency in the same way). This research has found that, under simple majority rule, legislation that is up to twice as costly as it is beneficial can pass, because half of the costs can be passed off to those outside the

[24]Altering the distribution of potential costs to be uniform from $\underline{C}$ to $\overline{C}$ (rather than 0 to $\overline{C}$) allows confirmation of Proposition 3 in the full model, as well.

coalition.[25] In this model, with the voting rule endogenously chosen, such inefficient legislation will not pass. To see this, recall that the costs only exceed the benefits for some $C > C_\beta$. However, no cooperation can be sustained for such a C, because the voting rule was chosen so that only a coalition of the whole can support cooperation at C_β. Therefore there is no coalition size that can maintain cooperation for any larger C; that is, cooperation would break down prior to the bills being more costly than beneficial. This finding yields Proposition 5.

Proposition 5: Given an optimal voting rule, no inefficient legislation ever passes.[26]

This is not to say that an optimal set of bills is passed each period. Indeed, when benefits exceed costs, more utility could be gained from including more bills in the coalition. This relationship is true even up to universalism. However, given the uncertainty of bill costs, the voting rule is likely chosen either to not include enough bills or to be too strict to allow sustainable cooperation over any legislation. In other words, much efficient legislation does not pass. This finding yields Proposition 6.

Proposition 6: Given an optimal choice of voting rule and uncertainty about future bill costs, much efficient legislation does not pass.

Propositions 5 and 6 have an interesting implication for how we should view legislative behavior. Optimally, the voting rule that guarantees universalistic passage of all beneficial bills would be chosen. This would happen if C were fixed or known prior to legislators choosing the voting rule. With uncertainty, legislators face the possibility of choosing V too high to allow *any* cooperation or so low as to encourage *too few* pieces of legislation. They take a compromise position based on the expected distribution of costs. No single voting rule can allow optimally sized coalitions for all types of legislation. However, the imperfect voting rule chosen is still the best choice, because enough bills pass at moderate costs to more than compensate for the ones not passed at low or high costs.

[25] There is occasionally a distinction drawn between economic efficiency and political efficiency (Niou and Ordeshook 1985). Here we are referring only to political efficiency, concerning the political costs and benefits felt by the model's legislators.

[26] When costs are allowed to vary across individuals, this result changes slightly. Logrolls as a whole are always efficient, but particular bills within a logroll can be inefficient. Inefficient bills are only included when all efficient bills have been added to the logroll, but additional members are still needed to ensure that cooperation is sustained via an oversized majority. Proof is available from the authors upon request.

Thus, when we observe a legislature not passing legislation that most would consider beneficial we should not necessarily see that as a bad outcome. From a longer-run perspective, that event might only be signaling that the legislature is actually working efficiently. But if such conditions occur continually, legislators may wish to change their voting rules.

As a final note, Proposition 5 is inconsistent with Baron's (1991) proof that inefficient legislation can pass in equilibrium. We find a different result because we allow an endogenous choice of the voting rule. In our model, the chamber attempts to choose a voting rule that is as inclusive as possible, while still maintaining cooperation over efficient bills. In Baron's model, the voting rule is assumed to be majority rule even when a more inclusive rule could pass legislation. Thus, in our model cooperation will break down before costs make the legislation inefficient (where $C > B$), whereas in Baron's model cooperation can be maintained for quite inefficient bills (with costs up to twice as large as benefits).[27]

This difference is not as stark as it might seem. If we fixed the voting rule at simple majority and varied the parameters of the model, we could create the same predictions that Baron does. Thus, what our finding really suggests is that inefficient legislation should only arise as a chamber's institutions become ill suited for an evolving reality (i.e., new sets of parameters).

Extensions

Before concluding, there are three extensions of our model worth discussing: more fully endogenizing the voting rule; endogenizing parties; and applying our model to government formation processes. The first relaxes our assumption that the voting rule is fixed after it is chosen; the second discusses how other important institutional features might be endogenized; and the third is a simple empirical application.

Allowing the Voting Rule to Vary by Period

While our model allows the voting rule to be endogenously chosen, it does not allow the voting rule to be changed over time. Depending on how uncertainty over bill passage is operationalized, the solution may vary somewhat if legislators are allowed to alter the voting

[27] Again, allowing the bundling of legislation, for example via an omnibus bill, would make the passage of inefficient legislation easier in our model.

rule as the game progresses. Here, we explore three possible ways to allow the voting rule choice to be revisited.

First, assume that the voting rule is chosen each period before any coalitions are formed and that bill cost is initially unknown, but fixed ever after. Outside of the first period, the voting rule will be chosen knowing the cost. Since legislators want to maximize the number of good bills passed and minimize the number of bad bills, there are two possible outcomes. Either bills are net beneficial, in which case the voting rule will be chosen to ensure that a universal coalition of bills will pass, or bills are net costly, in which case the voting rule will be chosen to ensure that no bills pass. Of course, this degenerative result holds because the voting rule is being set with perfect knowledge of what all future bills will be worth. The answer changes if some level of uncertainty is introduced.

Now assume that the voting rule is still chosen before coalitions are formed, but that new costs are drawn each period from some distribution. Under these conditions, the legislators can no longer anticipate future costs with perfect foresight. As a result, the decision over choosing a voting rule looks the same each period, and thus allowing it to change is logically equivalent to holding it constant after the first period. It can be shown that the comparative statics in this setting are identical to those formally derived above. Once again, each period the legislators would adjust the voting rule such that universal coalitions of beneficial bills pass but no net costly bills pass.

Finally, assume that new costs are drawn each period, but now the chamber has priors over who will be in the next coalition. Under these conditions, each legislator has a short-run incentive to set the voting rule to maximize one's own expected utility. The more bills that are passed, the higher the likelihood of getting one's bill passed, but also the more each legislator has to pay for other legislation. Individuals with different priors are likely to have different optimal choices. Thus, the voting rule chosen should depend upon the voting rule already in place, the distribution of priors, parameter values such as bill cost and discount rates, and the constitutional majority necessary to change the voting rule. It is beyond the scope of this paper to actually solve for the conditions under which the voting rule would be stable or would change, but clearly, one would expect larger constitutional majorities to make voting rules more stable.

Endogenizing Parties

What role might political parties play in our legislative setting? After all, there does not exist a legislative chamber in which parties do not operate. By starting with a focus on individuals, we are not meaning to suggest that parties are irrelevant. Rather, we are following the lead of such party experts as Aldrich, Rohde, Cox, and McCubbins, by creating a legislative chamber based upon individual incentives from which one could derive the kinds of influences that eventually will lead to party formation.

Aldrich uses the individual politician as the unit of analysis in asking the question, "Why Parties?": "My basic argument is that the major political party is the creature of the politicians, the ambitious office seeker and office-holder. They have created and maintained, used or abused, reformed or ignored the political party when doing so has furthered their goals and ambitions. The political party is thus an 'endogenous' institution—an institution shaped by these political actors" (1995, 4).

Likewise, Cox and McCubbins note: "[A] theory of the *internal organization* of parties . . . must begin with individual politicians and their typically diverse preferences, explaining why it is in each one's interests to support a particular pattern of organization and activity for the party. Accordingly, we begin not with parties and postulated collective goals but rather with legislators and postulated individual goals" (1993, 108, emphasis in original).

The model we have created is highly conducive towards understanding party formation. As cooperation gets more difficult, because of legislature size (N), coalition formation difficulties (δ), or bill costs (C), parties could be an excellent tool towards maintaining bill passage. If parties are collections of individuals with similar preferences, then cooperation will be facilitated because members of the same party will want the same bills or types of bills passed. Alternately, if parties are simply coordination devices (i.e., groups of legislators who agree to support one another's legislation and exclude others) then within-party punishment strategies could help reinforce otherwise unsustainable cooperation. Analysis of these scenarios could answer questions such as: (1) what size parties will form; (2) how many parties will there be; and (3) when would we expect partisan cooperation to break down?

However, before analyzing these issues, scholars must have a conceptual framework in which to analyze them. This paper provides that framework, as well as analyses of voting rule choice, of the tradeoffs between universalistic and minimum winning incentives, and of how these factors influence coalition formation. Having done that, the groundwork is now laid for a future analysis of parties, as well as other important institutional features such as committees. One benefit of this approach lies in the ability to contrast the predictions of partisan behavior arising within the framework provided here to those of the commonly studied unidimensional preferences approach. It

also will be interesting to see exactly how a chamber, with multiple endogenous institutional tools at its disposal, chooses among them. What are the tradeoffs between voting rules and parties? When will one institution be changed and when will another? While these are certainly questions worth answering, we must start with one institution and work our way up. In our view, the voting rule is among the most fundamental of institutional features in a legislative chamber and thus is studied first.[28]

An Application to Government Formation

Having said that, a brief discussion of government formation is feasible here. Assume for the moment that each of the *N* actors is not a legislator, but rather a party. Assuming that some internal, here unspecified, mechanism exists that ensures party discipline, we can then think of this model as a government formation model with parties as the unit of analysis. Interestingly, our coalition formation predictions of this model are consistent with extant theories of surplus majority governments. Luebbert (1986, 79) argues that "oversized" governments can be created to prevent one party from being able to bring down a government. That is, extra parties, ones that are not needed to have enough votes to ensure that a bill will pass, are included so that even if one party defects from the governing coalition, that government will still have enough votes to ensure passage of the legislation under consideration. This is equivalent to our result that minimum necessary coalitions (MNCs) will be chosen in excess of minimum winning coalitions (MWCs).

A current example of such an effort is Ehud Barak's attempts to form a government in Israel with both Shas and Likud. Only one is needed to have a parliamentary majority, but Barak wants both parties to help ensure that the peace process moves forward (*Financial Times*, 5/26/99). Such reasoning is consistent with our model's predictions of oversized coalitions. When the costs of the legislation to be proposed are high (and the peace process certainly is controversial within Israel), the proposer wants extra members in the coalition to ensure that the policy logroll is sustained.

Conclusions

Returning to the initial question, what does this study teach us about how a foresighted legislative chamber will design its institutions in response to *ex ante* incentives for universalism and *ex post* incentives for minimum winning coalitions, and what coalitions will form as a result? First, we find that institutions are not necessarily going to be designed to enforce universalism. Rather, the voting rule is set such that anything from minimum winning to universal coalitions are possible. This choice is made even though a voting rule *could* be chosen to ensure that universalistic outcomes always occurred. Such a choice is not made because many bills that otherwise could have been passed (higher cost but still net beneficial ones) would not pass as a result.

Second, and on the flip side, we find that oversized coalitions are formed even though there is an incentive to create minimum winning coalitions. Proposers sometimes find themselves constrained to propose oversized coalitions because cooperation over a minimum winning coalition would not be sustained. That is, because it is easier to maintain cooperation when a coalition has extra members, a proposer will propose a coalition that has just as many extra members as necessary to ensure that the logroll will succeed. Thus, what we find is that, rather than predicting minimum winning coalitions, this model predicts "minimum necessary coalitions" (i.e., the smallest necessary to maintain cooperation) and that universalism occurs when the minimum necessary coalition is the chamber of the whole.

Interestingly, the same features that lead to larger coalitions through a more inclusive voting rule also lead to fewer extra members in the coalitions (i.e., closer to minimum winning). Larger legislatures with more costly legislation and lower discount values are likely to be drawn to more universalistic coalitions for a given voting rule. They are also more likely to be unable to sustain cooperation. Thus, these legislatures are more likely to adopt less inclusive voting rules and more oversized coalitions.

Of course, this study is only a starting point. There are numerous interesting and important legislative features still to be analyzed. For example, parties, committee systems, and the ability to create omnibus bills all affect the ability of legislators to form and maintain logrolls. Both parties and committee systems are alternative institutional structures that can be established *ex ante* to help encourage universalistic outcomes. Conversely, omnibus bills are actually ways of ensuring that all the bills of a given logroll are passed with one vote. Thus, all three of these features can affect the ability of a chamber to pass legislation. It should prove particularly interesting to examine, both theoretically and empirically, how a legislative chamber that has the option of setting combinations of institutions, such as a voting rule and a committee structure, will organize.

[28]Of course, one can assume that each *N* is a party rather than an individual. As a result, we would have a model with parties, albeit all of equal size, rather than individuals as the unit of analysis and all the results would follow.

Appendix

A. Basic Game, Unanimity Rule

The analysis throughout the paper and its appendix focuses on the individual with the greatest incentive to defect from the cooperative strategy. In this case, that is the legislator whose bill comes up first. She considers voting for her own bill and then voting against her coalition members' bills.

Her cooperative payoff is $\frac{B-C}{1-\delta}$, that of being in the unanimous coalition forever, with each turn's payoff discounted by δ. Her defect payoff is $B-\frac{C}{N}$, that of receiving her bill this period and having no further bills pass. For the logroll to be in equilibrium, the cooperative payoff must exceed the defect payoff, or $\frac{B-C}{1-\delta} \geq B-\frac{C}{N}$. Solving for δ yields $\delta \geq \delta_U^* = \frac{C-C/N}{B-C/N}$.

B. Basic Game, Nonunanimity Rule

As above, we are looking at the legislator whose bill is first on the docket. Her cooperative payoff is $B-VC + \frac{\delta}{1-\delta}\big(V(B-VC)+(1-V)(-VC)\big)$. Her defect payoff is $B-\frac{C}{N}-\frac{\delta}{1-\delta}(VC)$. Equilibrium logrolling requires $B-VC + \frac{\delta}{1-\delta}\big(V(B-VC)+(1-V)(-VC)\big) \geq B-\frac{C}{N}-\frac{\delta}{1-\delta}(VC)$.

Solving for δ yields $\delta \geq \delta_{NU}^* = \frac{VC-C/N}{VB+VC-C/N}$. Solving for V yields $V \leq V^* = \frac{C}{N\left(C-\frac{\delta}{1-\delta}B\right)}$, as the range of voting rules under which cooperation may be sustained.

C1. Limits of Cooperation under Minimum Winning Coalitions

If the coalition size is minimum winning, cooperation can be sustained over the region defined in Appendix B. That is, cooperation if: $\delta \geq \delta_{NU}^* = \frac{VC-C/N}{VB+VC-C/N}$. In this expanded model, C is chosen randomly. We can see the range of C for which cooperation is sustainable under a minimum winning coalition by solving for C: $C \leq B\left(\frac{\delta}{1-\delta}\right)\left(\frac{VN}{VN-1}\right) \equiv C_\alpha$. As long as C is drawn less than this cutpoint, cooperation is sustained with a minimum winning coalition.

C2. Limits of Cooperation under Oversized Coalitions

With an oversized coalition, cooperation is easier to maintain. Consider the incentives of those coalition members most likely to defect. Because the coalition is oversized with A additional members ($S = VN + A$), the legislation will pass unless $A + 1$ members defect. After the first $A + 1$ bills have passed, these members consider defection. If they all defect, the next $VN - 1$ bills will not pass, but they will be left in bad standing for the indefinite future. Depending on the value of the future, defection could be more beneficial than cooperation. But defection is an individual decision, rather than a collective one. The key question is will each individual want to defect given the other A members are defecting?

We solve for the conditions under which, if A others are defecting, the last member of the defecting group would go along with them instead of cooperating with the coalition. This is a subgame perfect equilibrium. If this individual cooperates with the coalition, the next $VN - 1$ bills pass, and she has a $\frac{VN}{N-A}$ chance of being in the future coalitions of size VN.[29] Thus the forward-looking cooperative payoff is: $-\frac{VN-1}{N}C + \frac{\delta}{1-\delta}\left(B\frac{VN}{N-A}-VC\right)$.

The forward-looking defect payoff is being out of good standing indefinitely, with minimum winning coalitions still producing legislation: $-\frac{\delta}{1-\delta}VC$.

[29] The pool of people in good standing in the future would be $N - A$ because A people are defecting this period under this scenario. Future coalitions are of size VN because, if this individual is willing to cooperate with the coalition with A people going into bad standing, a similarly placed first member looking at $VN - 1$ bills in the future would go along with the minimum winning coalition. That is, this individual is facing *identical incentives* to the future first member of a minimum winning coalition with A people in bad standing. If a $VN + A$ coalition is sustainable this period, a VN coalition is sustainable in the future with A people in bad standing. The chance of each of those $N - A$ people in good standing being in this coalition of VN members is simply $\frac{VN}{N-A}$. Notice that none of these new defectors is needed to form the coalition of VN members, so they are not brought back into good standing under parole.

Thus cooperation is sustainable iff:

$$-\frac{VN-1}{N}C+\frac{\delta}{1-\delta}\left(B\frac{VN}{N-A}-VC\right)\geq-\frac{\delta}{1-\delta}VC.$$

$$\Leftrightarrow\frac{\delta}{1-\delta}\frac{BVN}{N-A}\geq\frac{VN-1}{N}C$$

$$\Leftrightarrow\frac{\delta}{1-\delta}\geq\left(\frac{VN-1}{N}\right)\left(\frac{N-A}{VN}\right)\left(\frac{C}{B}\right)$$

$$\Leftrightarrow\delta\geq\delta^{*}=\frac{\left(\frac{VN-1}{N}\right)\left(\frac{N-A}{VN}\right)\left(\frac{C}{B}\right)}{1+\left(\frac{VN-1}{N}\right)\left(\frac{N-A}{VN}\right)\left(\frac{C}{B}\right)}$$

This equation can be solved for A to find the smallest oversized coalition that can still sustain cooperation for any given value of C:

$$A\geq A^{*}=N-\left(\frac{\delta}{1-\delta}\right)\left(\frac{VN}{VN-1}\right)\left(\frac{BN}{C}\right).$$

Substituting $A = 0$ shows the limit where the minimum winning coalition is no longer sustainable:

$$A=0\Leftrightarrow C\leq B\left(\frac{\delta}{1-\delta}\right)\left(\frac{VN}{VN-1}\right)\equiv C_{\alpha}.$$

This reconfirms the result from Appendix C1.

Substituting $A = N - VN$ shows the limit where even a universal oversized coalition is no longer sustainable:

$$A=N-VN\Leftrightarrow C\geq\left(\frac{\delta}{1-\delta}\right)\left(\frac{NB}{VN-1}\right)\equiv C_{\beta}.$$

D. Optimal Choice of Voting Rules in Expanded Model

The expected value based on the choice of V is generated assuming C is drawn from a uniform distribution between 0 to $\overline{C}$. In this paper, we assume that $\overline{C}$ is sufficiently large that it is not constraining (that is, $\overline{C}>C_{\beta}\forall V$).

Depending on the value of C chosen, the proposer will form a minimum winning coalition (size VN), or the smallest oversized coalition (size $VN + A$) that will allow cooperation. The expected utility is derived by integrating under the curve illustrated in Figure 4 for the probability of drawing any given value of C. That figure shows the size of coalitions sustainable, which is multiplied by their probability of occurring and by $\frac{\delta}{1-\delta}(B-C)$ to generate the expected utility.

Overall expected utility is thus equivalent to the utility achieved from a coalition of size VN evaluated from $C = 0$ to $C = C_{\beta}$ added to the expected utility from the additional A members evaluated from $C = C_{\alpha}$ to $C = C_{\beta}$:

$$\begin{aligned}EU&=EU_{VN}+EU_{A}\\&=\int_{0}^{\left(\frac{\delta}{1-\delta}\right)\left(\frac{NB}{VN-1}\right)}\frac{\delta}{1-\delta}VN(B-C)\frac{1}{\overline{C}}dC\\&\quad+\int_{\frac{\delta VBN}{(VN-1)(1-\delta)}}^{\left(\frac{\delta}{1-\delta}\right)\left(\frac{NB}{VN-1}\right)}\frac{\delta}{1-\delta}A(B-C)\frac{1}{\overline{C}}dC\end{aligned}$$

Substituting in $A=N-\left(\frac{\delta}{1-\delta}\right)\left(\frac{VN}{VN-1}\right)\left(\frac{BN}{C}\right)$, the boundary condition from Equation 5, yields

$$\begin{aligned}EU=&\int_{0}^{\left(\frac{\delta}{1-\delta}\right)\left(\frac{NB}{VN-1}\right)}\frac{\delta}{1-\delta}VN(B-C)\frac{1}{\overline{C}}dC\\&+\int_{\frac{\delta VBN}{(VN-1)(1-\delta)}}^{\left(\frac{\delta}{1-\delta}\right)\left(\frac{NB}{VN-1}\right)}\frac{\delta}{1-\delta}\left(N-\left(\frac{\delta}{1-\delta}\right)\left(\frac{VN}{VN-1}\right)\left(\frac{BN}{C}\right)\right)(B-C)\frac{1}{\overline{C}}dC\end{aligned}$$

Working out these integrals gives the expected utility from the game. We wish to maximize this expected utility with respect to V. To do so, we solve $\frac{\partial EU}{\partial V}=0$. This gives the implicit solution for V^*:

$$-\ln(V^{*})=N-V^{*}N+1-\frac{\delta}{1-\delta}\frac{2N^{2}-V^{*}N^{2}+2V^{*}N-N}{2(V^{*}N-1)}.$$

Due to the transcendental nature of this function, we cannot solve explicitly for V^*. Further results detailed in the text come from a computer simulation. Comparative statics equations can be generated through implicit differentiation. In particular,

$$\frac{\partial V^{*}}{\partial N}=\frac{-V^{*}\left(2+NV^{*2}(4-N(\delta-2)-4\delta)+2N^{2}V^{*3}(\delta-1)+\delta(4N-3)-2V^{*}(1-N(\delta-2)-2\delta+N^{2}\delta)\right)}{2\left(1-\delta-N^{2}V^{*}(3V^{*}\delta-3V^{*}+\delta)\right)+N^{3}V^{*}(V^{*2}\delta-V^{*2}+\delta)+NV^{*}(4\delta-3)}$$

$$\frac{\partial V^{*}}{\partial\delta}=\frac{V^{*}N(V^{*}N-1)(1-2V^{*}+N(V^{*}-2))}{2(\delta-1)(1-\delta-N^{2}V^{*}(3V^{*}\delta-3V^{*}+\delta))+N^{3}V^{*}(V^{*2}\delta-V^{*2}+\delta)+NV^{*}(4\delta-3)}$$

The signs of these equations are determined by looking at the values for V^* generated above. These are illustrated in Table 1 through values generated by a computer simulation.

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